Teaching Natural Deduction to improve Text Argumentation Analysis in Engineering Students

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Abstract. Teaching engineering students courses such as computer science theory, automata theory and discrete mathematics took us to realize that introducing basic notions of logic, especially following Gentzen's natural deduction approach [6], to students in their first year, brought a benefit in the ability of those students to recover the structure of the arguments presented in text documents, cartoons and newspaper articles. In this document we propose that teaching logic help the students to better justify their arguments, enhance their reasoning, to better express their ideas and will help them in general to be more consistent in their presentations and proposals.

Keywords: education, learning, logic, deductive reasoning, natural deduction.

1 Introduction

One of the interesting capabilities when reading a paper or analysing a document is the ability to recover the underlying structure of the arguments presented. Studying logic and specially Gentzen's natural deduction approach gives the student that reads a paper, the news, or an article in a magazine, the ability to recover those facts that support the conclusions, the arguments, the basic statements and the distinction among them. That is, to distinguish between the ones that the author assumes to be true and the ones proposed and justified by himself/herself. The experience obtained teaching logical concepts to our students, in courses such as automata theory, theory of computation and artificial intelligence at different universities and institutes, showed us that it may help the student to see the structure of the argumentation presented by the author, which brings insights on how to write a document, to support judgements and defend a position or point of view, which are important capabilities of any undergraduate student. Our proposal for teaching natural deduction to undergraduate students in their first year may have an important impact in their professional studies, improving their analysis and synthesis abilities as well as allowing for abstract reasoning.

An empirical-inductive research was accomplished. The research question was: How is it possible to measure the improvement in the analysis and synthesis in a uniform way under the context of a student reading argumentative texts? The strategy consisted in combining the learning of logical concepts with the reading of news on the paper, political cartoons and articles of different types. The student was confronted to the following questions: (1) Which is the impact of the article (cartoon, etc.)? (2) Identify the propositions which represent the conclusions of the document; (3) Which propositions are assumed to be true (identify the premises)? (4) How does the author goes from premises to conclusions? Which kind of arguments are explicit or implicit in the text; (5) Are the conclusions well supported? Are they logical consequences?

In the following sections a revision of logic concepts will be introduced, the natural deduction approach and its application in the analysis of structure recovery from sample texts will be presented.

2 Logic and Language

Logic is the study of valid *arguments* [1, 2]. An argument consists of a set of statements called *premises* and an assertion called *conclusion*. For example:

> *Maggie is in the Library or in the coffee bar* … Premise
> *Maggie is not in the coffee bar* … Premise
> --
> *Therefore, Maggie is in the Library* … Conclusion

We say that an argument is *valid*, when true premises would **never** produce a false conclusion. Valid arguments are important as they define *rules of inference*. These rules allow us to obtain conclusions from certain premises. Whenever the premises are true, a rule of inference generate only true conclusions. Rules of inference have the important property of preserving truth.

Consider the following example:

> If Sandy has hepatitis then her skin colour will become yellow
> Sandy has a yellow colour skin
> --
> Therefore, Sandy has hepatitis

In this example the argument is not valid. There are many reasons why the skin of Sandy may have a yellow colour (maybe she using makeup), without having hepatitis. In this case deducing that Sandy has hepatitis from the fact that she has yellow colour skin is not a logical deduction.

In essence, the main property of a valid argument is that true premises just produce true conclusions.

A sequence of statements, called propositions, is combined in texts to build proofs. A proof is a sequence of ordered propositions, the last of them is called conclusion. These propositions have the property that each of them can only be a premise, the conclusion or the result of applying an inference rule on previous propositions.

The idea would be to introduce topics of deductive reasoning in the curricula of bachelors of engineering.

In the following section an introduction to natural deduction is presented.

2.1 Natural Deduction

Natural deduction is a logical system that reflects patterns that people use when reasoning [1, 3]. In this system the meaning of each logical operator is defined according to the way it is found in natural language discourse.

Logic comes from the analysis of the way in which human beings use language; the way in which people connects sentences using linguistic operators such as "or", "and", "not" and the conditional "if … then." Each logical operator has associated some inference rules. These inference rules are of two types*: introduction rules* and *elimination rules*. The introduction rules link two propositions using a particular operator and the elimination rules tell us when it is possible to eliminate such operator. In the following subsections the inference rules for the logical connectives will be introduced.

2.1.1 The meaning of AND

Let's have a look to the way we use the conjunction operator "and" in English through some example sentences:

(i)

Mary was born in California
Peter works in New York

Therefore: *Mary was born in California* **and** *Peter works in New York*

The argument (i) shows that if we assume that both sentences are true, then the concatenation of both using the "and" will be as well true.

(ii)

Larry loves basketball **and** *Peter likes playing squash*

Therefore: *Peter likes playing squash*

In example (ii) we assert that if the concatenation of two sentences with the word "and" is true, then any of the sentences in the construction will be true by itself.

Generalizing both arguments (i) and (ii) by replacing the sentences for propositional variables P and Q respectively, it is possible to define the rules that allow the introduction and elimination of the “and” connective:

AND-Introduction

```
  P     Q
--------------
 P AND Q
```

AND- Elimination

```
P AND Q        P AND Q
-------------  -------------
    P              Q
```

2.1.2 The meaning of OR

The word “or” in English is commonly used in an exclusive form:

John is in the kitchen **or** *she is taking a shower*

This means that just one of the options can be true. The exclusive “or”, XOR, is expressed in the following way:

P	Q	P XOR Q
F	F	F
F	T	T
T	F	T
T	T	F

This definition of “or” presents the problem that it does not work in a compositional way. So George Boole preferred to provide an inclusive definition that allows for the possibility of taking both as true.

P	Q	P OR Q
F	F	F
F	T	T
T	F	T
T	T	T

So, the disjunction of two sentences is considered to be true if any of the sentences is true or both of them are true.

To introduce the inference rules for the OR connective in logic, it is important to analyse the following argument:

(iii)

Peter study chemistry

Therefore: *Peter study chemistry* **or** *the sky is green*

If we know that a sentence is true, we can add any other sentence without any consideration about its truth or falsity using the disjunctive word "or" and the resultant sentence will preserve the truth.

(iv)

(a) *Larry practices Judo or he practices Kung-fu*
(b) *If Larry practices Judo, he is dangerous*
(c) *If Larry practices Kung-fu, he is dangerous*

Therefore, we may conclude that: *Larry is dangerous*

In argument (iv) we see that there are two sub-arguments in the construction. One which concludes that *Larry is dangerous* based on the fact that *he practices Judo*, and another, which asserts that *Larry is dangerous* knowing that he *practices Kun-fu*. We see in this example that both sub-arguments arrive to the same conclusion, and as we had considered all the sentences that are part of the "or" sentence, we are able to conclude that it is possible to derive that *Larry is dangerous* based on the fact that all the possibilities of the "or" conclude to the same sentence.

Generalizing both arguments (iii) and (iv) by replacing the sentences for propositional variables P and Q respectively, it is possible to define the rules that allow the introduction and elimination of the "or" connective:

OR-Introduction

```
   P
-----------
P  OR Q
```

OR- Elimination

```
                  P        Q
                ------   ------
P   OR  Q         R        R
--------------------------------------
                R
```

2.1.3 Meaning of the conditional

Some examples of the use of conditional in natural language are the following:

(v)

Mark hits Peter on the face

Therefore: *Peter has pain*

Therefore: *if Mark hits Peter on the face then Peter has pain*

In argument (v) we assume that we have a proof that from: *Mark hits Peter on the face* we can derive that *Peter has pain*. This conclusion can be expressed in the form of a conditional.

(vi)

a) *If Mark hits Peter on the face then Peter has pain*
b) *Mark hits Peter on the face*

Therefore: *Peter has pain*

Argument (vi) shows a typical structure in natural language.

Generalizing both arguments (v) and (vi) by replacing the sentences for propositional variables P and Q respectively, it is possible to define the rules that allow the introduction and elimination of the conditional connective:

→-Introduction

$$\dfrac{\dfrac{P}{Q}}{P \rightarrow Q}$$

→- Elimination (MODUS PONENS MP)

$$\dfrac{P \rightarrow Q \quad P}{Q}$$

The introduction rule says that if from supposing P to be true, you may conclude Q, then it is possible to assert "if P then Q." There are many examples in natural language of this construction, such as: "If I fall down the bridge, I will kill myself", "if the loan is low, I won't be able to buy the house", etc.

2.1.4 Contradiction

The contradiction has not a counterpart word in natural language. The contradiction is expressed using the symbol, ⊥**,** and has an important role in mathematics. Finding a contradiction in a system, collapses the system as everything becomes provable.

⊥-Introduction

$$\dfrac{P \quad \text{AND} \quad \text{NOT } P}{\bot}$$

⊥- Elimination

$$\dfrac{\bot}{R}$$

The introduction rule says that if we have a proposition and its negation then we have a contradiction. Example: "the battler is guilty and the battler is not guilty", "the Earth is round and the Earth is not round." These sentences are always false.

The rule for elimination says that if there is a contradiction then everything is provable.

2.1.5 Negation NOT

The last connective is the negation NOT. In this section we present the classic form of negation, which means that if from assuming a sentence to be true we arrive to a contradiction, then we know that its negation is true.

NOT-Introduction **NOT – Elimination (DOUBLE NEGATION)**

```
   P                        NOT  NOT  P
  -----                   -----------------------
   ⊥                              P
----------------
  NOT  P
```

Example:

```
Let's assume:          Bush is a defender of human rights
                      ----------------------------------------------------
                      Therefore:   Bush attacks terrorists as they hurt
                                   many innocent people
                      -----------------------------------------------------------
                      Hence:    Bush invades countries to catch terrorists
                     ------------------------------------------------------------------
                     But when invading a country:    Bush hurts innocent people
                     ----------------------------------------------------------------------
                                  Therefore: Bush is a terrorist
                 --------------------------------------------------------------
                 This means that:          Bush is a terrorist and Bush is a defender
                                                  of the human rights
                     -----------------------------------------------------------------
                                  Which is a contradiction:     ⊥
    -----------------------------------------------------------------------------------
       Conclusion:   It is not true that Bush defends the human rights
```

As we mentioned before, the introduction rule is representing classical negation. We should not confuse this rule with the reductio ad absurdum rule or proof by contradiction rule:

Classical Negation Reductio ad Absurdum (RAA)

```
        P                                NOT  P
```

-----   -------------
⊥   ⊥
----------------------   ----------------------
NOT P   P

While classical negation is accepted by all logical systems, reductio ad absurdum is not. The intuitionistic logic, base of constructivism, does not accept the RAA rule.

2.2 Example

An ancient Zen koan called "Ganto's Axe" says (taken from [4]):

> One day, Tokusan said to his disciple Ganto, "Those two monks have been with me for many years. Go and make them an examination". Ganto took the axe and went into the room where those monks where preying. He lifts up the axe over their heads and said: "if you say a word I will cut your heads; and if you don't say a word, then I will cut your head."

Anybody who hears that sentence realizes that Ganto is going to cut the monks heads. But how can we arrive to such a conclusion? Is the argument valid? In order to answer these questions let's try to formalize the reasoning.

There are two propositions:

A = the monks say a word
B = Ganto cuts both monks head

There are three premises in the argument;

P1. The monks say a word or they do not say a word: A OR (NOT A)
P2. If the monks say a word, Ganto will cut their heads: A → B
P3. If the monks do not say a word, Ganto will cut their heads: (NOT A) → B

Suppose that the monks say a word: A …S1

Applying Modus Ponens, from P2 and S1, we obtain: B …C1

Suppose that the monks say nothing: NOT A …S2

Applying Modus Ponens, from P3 and S2, we obtain: B …C2

Hence, applying the OR-ELIMINATION Rule,
from P1, S1-C2, S2-C2 we conclude: B

Which means, that Ganto will cut the monks head.   **¡What we wanted to prove!**

The formalization of the argument allows confirming that our original intuition is valid. It is to say that it actually follows from the original premises using the rules of inference.

It may look that it is just a lot of work to arrive to an obvious conclusion, but science and history had shown to human kind that many of our "obvious" initial conclusions (such as; "the Earth is flat", "the Sun is the centre of the universe", "there are only four elements: air, water, fire and soil"), were actually wrong.

But, Ganto's axe koan has not finished yet and continues:

> Both monks continued their meditation as if they had heard nothing. Ganto lift down the axe and said: "You are authentic Zen disciples," He returned and told Tokusan what happened. "I Know what you mean, Ganto" but tell me: "What is what you mean?" "Tokusan must admit them," said Ganto, "but they should not be admitted by Tokusan."

The conclusion of the koan gave us the feeling that even recognizing that our formal system captures basic aspects of our reasoning, human mind (fortunately) is still far ahead from our understanding of it.

In the following section the expressive power of our logic language will be improved with the addition of quantifier symbols.

2.3 Introducing Quantifiers

In the language defined in Section 2.1, it is not feasible to express arguments such as:

a) *All Greeks are philosophers*
Socrates is Greek

Therefore: *Socrates is a philosopher*

b) *Socrates is Greek*
Socrates is a philosopher

Therefore: *Some Greeks are philosophers*

To be able to express reasoning such as those shown in a) and b) requires replacing our definition of proposition for other that allows us to introduce details within the proposition itself. This is accomplished by including predicates within the language as well as a set of constants and variables that will represent individuals in the propositions. Let's analyse the following sentence:

Socrates is Greek

This is a proposition about which we can say it is true or false. What happens if some of the components become abstract entities:

Socrates is Greek
--------------------- Replacing Socrates for an abstract individual
X *is Greek*

Now we have a structure for the adjective to be Greek, we say that Greek is a predicate that requires of an individual to become a proposition: X is Greek or, if we use the more standard syntax, greek(X). With this method we can extend our language to terms like the followings:

greek(Socrates) --- Socrates is Greek
loves(X, Y) --- X loves Y
loves(John, Rose) --- John loves Rose

Now let's use our logical operators defined in Section 2.1:

man(John) & woman(Rose) & loves(John, Rose)
greek(X) $\rightarrow$ *philosopher(X)* --- if X is Greek, then X is a philosopher
…

If we introduce the **for all** and **there exists**, then we complete our idea:

$\forall$*x.(greek(x)*$\rightarrow$ *philosopher(x))* ---- all Greeks are philosophers
$\exists$*x.(women(x) & loves(John,x))* ---- there exists a woman X that John loves

Now, let's introduce the rules that will allow us to reason with quantifiers:

Rules for the universal quantifier $\forall$**(for all)**

$\forall$-Introduction

P(a) *

Q(a)

$\forall$x.(P(x)$\rightarrow$ Q(x))

$\forall$-Elimination (UNIVERSAL INSTANTIATION UI)

$\forall$x.P(x)

P(a)

***** "a" should not appear in the premises

Rules for the existential quantifier ∃ **(there exists)**

∃-Introduction

$$\frac{P(a)}{\exists x.P(x)}$$

∃-Elimination

$$\frac{\exists x.P(x) \qquad \begin{array}{c} P(a) \\ \hline R \end{array}}{R^{**}}$$

****** R most not contain an "a"

2.4 Some examples

In deductive reasoning, contrary to inductive reasoning, a specific solution is obtained from a general rule. For example, to obtain the area of a specific rectangle the general formula "Area = Basis × Height" is used with the corresponding substitution of variables for the actual values.

The following is an example of deductive reasoning:

Example 1. If we assume that the following statements are true

a) All men that wear a hat are bold.
b) Some men wear a moustache.
c) All bold men like red wine.
d) All men with moustache like beer.
e) Peter is bold.

Which of the following propositions is a valid conclusion?

f) Peter likes red wine.
g) Peter wears moustache.
h) Peter likes beer.
i) Peter wears a hat.

In deductive reasoning one must be 100% certain, no doubts are allowed.

Since premise (c) says that all bold men like red wine and since premise (e) says that Peter is bald, we can infer with certainty that (f) is true. This example illustrates a valid argument.

Let's make the argument formal:

Premises

P1. ∀x. (wears_hat(x) → bold(x))
P2. ∃y. (man(y) AND wears_mustache(y))
P3. ∀z.(bold(z) → likes_red_wine(z))
P4. ∀w.(wears_mustache(w)→ likes_beer(w))
P5. bold(Peter)

Possible conclusions:

C1. likes_red_wine(Peter)
C2. wears_mustache(Peter)
C3. likes_beer(Peter)
C4. wears_hat(Peter)

In this example the conclusion is straight forward since from P3 and P5 we can conclude C1:

$$\dfrac{\dfrac{\forall z.(\text{bold}(z) \rightarrow \text{likes\_red\_wine}(z))}{\text{bold(Peter)} \rightarrow \text{likes\_red\_wine(Peter)}}\ \mathbf{UI} \qquad \text{bold(Peter)}}{\text{likes\_red\_wine(Peter)}}\ \mathbf{MP}$$

Another example of deductive reasoning is:

Example 2. John, Peter, Paul and James all live in the first floor of an apartment building. One of them is a manager, another is a programmer, one of them is a singer and the other is a teacher. Assuming that the following statements are true, answer the question WHO IS THE MANAGER?

a) John and Paul have breakfast with the singer.
b) The manager gives Peter and James "a lift" to work.
c) Paul goes to the football game together with the manager and the singer.

Premise (a) does not let us eliminate either John or Paul from being the manager. Nevertheless, premise (b) eliminates the possibility that either Peter or James be the manager. Also, premise (c) eliminates Paul from being the manager. Therefore, the only conclusion of which we can be 100% certain is that John is the manager.

Let's make the argument formal:

Premises

P1. ∃x. (singer(x) AND have_breakfast_with(John, x) AND have_breakfast_with (Paul, x))
P2. ∃y. (manager(y) AND gives_a_lift(y, Peter) AND gives_a_lift (y, James))
P3. ∃z.∃w. (manager(z) AND singer(w) AND goes_to_the_football_game_with(Paul, z) AND goes_to_the_football_game_with (Paul, w))
P4. ∀u.(singer(u) OR manager(u) OR programmer(u) OR teacher(u))
P5. manager(Paul) OR manager(James) OR manager(John) OR manager(Peter)

In this case we have:

manager(Paul) OR manager(James) OR manager(John) OR manager(Peter)

∃y. (manager (y) AND gives_a_lift (y, Peter) AND gives_a_lift (y, James))

Therefore: NOT manager (Peter) AND NOT manager (James)

So, we have: manager(Paul) OR manager(John)
∃z.∃w. (manager (z) AND singer(w) AND
goes_to_the_football_game_with (Paul, z) AND
goes_to_the_football_game_with (Paul, w))

Then: NOT manager (Paul)

Therefore: manager (John)

Next we give our comments about the impact that introducing these logical concepts as well as presenting a large number of examples taken from newspapers, cartoons or scientific articles will have on college students.

3 Experiences in Teaching

The experience that we have obtained by teaching logical concepts to our students both at Tecnológico de Monterrey, ITESM, and Universidad de las Américas, UDLA, is that these concepts enhance the capacity of the student to understand and solve problems, analyze texts, make judgments, write documents, etc. All these capacities are fundamental for any professional. In the experiment the main result was to observe that the control group stayed a bit behind the other groups in which an effort was made in order to motivate the students to the critical analysis of documents.

Other observations are in relation to common errors: the students normally try to "to prove" general cases by means of examples. It is important to know that examples do not prove the veracity of a general statement; they are only good for illustrative purposes. On the other hand, a counterexample is enough to prove the falseness of a statement.

In order to illustrate what we just said, lets take an example: the problem that consists in "prove that the frequency for placing orders in a store increases if the demand increases" is proven by students by verifying that the frequency for a specific value of the demand is larger than the frequency corresponding to a smaller value of the demand, instead of analyzing the formula and see that the demand is in the numerator of the formula for the frequency and therefore when the demand increases, the frequency increases as well. On the other hand, when one asks to prove the falseness of a general statement, few times it is seen that by showing a counterexample proves such falseness.

On the other hand, we have noticed that in courses where we cover subjects related to first order logic, students show improvements in their argumentations and reasoning. We have noticed that they clearly understand or identify some fallacies due to:

- Ambiguities such as *"I saw John with the glasses".* Who was wearing the glasses, John or me?

- Generalizations such as: *"A poor man robbed a young girl; therefore all poor men are robbers."*
- Justify their own mistakes based on someone else's mistakes, for instance: *"How can my father ask me not to smoke if he does it himself."*

Just like the examples presented, there are many others that illustrate the necessity for enhancing the capacity of our students in their ability for argumentation and reasoning.

4 Conclusions

In the present document we have offered some arguments to support the incorporation of logic concepts following the natural deduction approach to students in their first year of undergraduate education. A framework has been introduced and some examples were analysed to show the possibilities of the approach. Our proposal is that using some of our students time to revise basic concepts of deductive reasoning may contribute very importantly to better accomplishments in their future courses and activities by improving their capacities for analysis and synthesis and providing them with a mental structure that facilitates abstract reasoning.

We propose that we should include, if not in an ad-hoc class but within some appropriate course, subjects that teach the proper usage of:

a) The OR (disjunction), AND (conjunction), the → (conditional) and the NOT (negation).
b) The rules of inference that allow the introduction or elimination of those logic operators.
c) Proofs for trueness or falseness of statements.
d) Proofs of solution existence, that is, to prove that a solution for a problem exists without actually showing the solution.

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